SPATIAL MODELS OF LEGISLATIVE VOTING

By

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Chapter 1: Introduction

Overview

In this book I show how to use a simple spatial model of voting as a tool to analyze parliamentary roll call data. Each legislator is represented by one point and each roll call is represented by two points – one for Yea and one for Nay. On every roll call each legislator votes for the closer outcome point at least probabilistically. These points form a *spatial map* that summarizes the roll calls. In this sense a spatial map is much like a road map. A spreadsheet with all the distances between every pair of sizeable cities in the United States contains the same information as the corresponding map of the U. S. but the spreadsheet gives you no idea what the U.S. looks like.[1] Much like a road map, a spatial map formed from roll calls is a way of visualizing the political world of a legislature. The closeness of two legislators in the map shows how similar their voting records are and distribution of legislators shows what the dimensions are.

The number of dimensions needed to represent the points is usually small because legislators typically decide how to vote on the basis of their positions on a small number of underlying evaluative or *basic* dimensions. For example, in recent U.S. Congresses, we can easily predict how a "liberal" or a "conservative" will vote on most issues. These basic dimensions structure the roll call votes and are captured by the spatial maps.

In this chapter I develop a simple theory of spatial maps that I call the *basic space theory of ideology*. In subsequent chapters I use the theory to show how to construct and interpret the spatial maps that reflect it. The theory is based upon the work of many scholars in psychology, economics, and political science over the past 50 years and is a parsimonious tool for understanding the construction and interpretation of spatial maps.

I begin with some observations on theory and meaning. A spatial map is a picture and for it to be a summary it must have a *meaning* for the viewer and that meaning must flow from *a theory about the picture*. In this sense, "a picture is worth a thousand words." My point is a simple one. Anyone can construct a spatial map using the computer programs I discuss in subsequent chapters. But the maps are worthless unless the user understands both the spatial theory that the computer program embodies and the politics of the legislature that produced the roll calls. A practitioner must be able to stand before an audience of her peers and explain the meaning of the spatial map.

After discussing theory and meaning, the rest of the chapter lays out my theory of spatial maps.

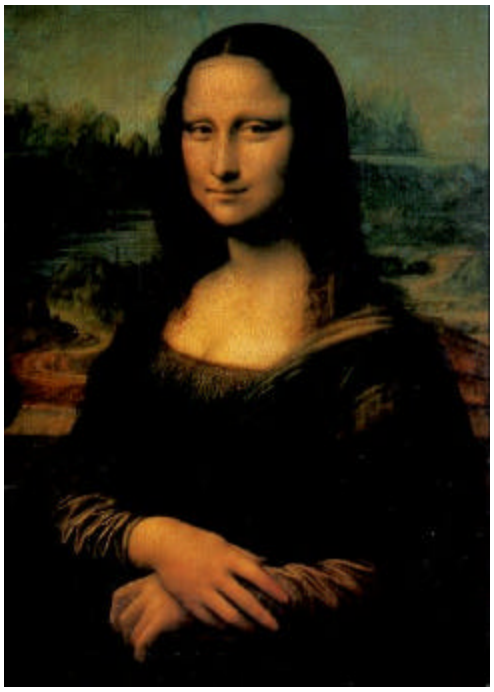
Theory and Meaning

To reiterate, this is a book about the use of pictures to summarize parliamentary roll call data. For the most part these pictures will consist of simple geometric representations of the legislators and of the roll calls. For example, members of the left or liberal party clustered together on one side of the picture and members of the right or conservative party clustered together on the opposite side of the picture. In this case a glance at the picture by an *experienced* researcher shows what the main sources of

conflict are within the parliament and how the roll call voting is structured. By "experienced" I mean that the researcher must understand how the picture was constructed and must understand the political environment of the parliament or legislature. It is the researcher's understanding of the theory about the picture that gives the picture meaning. Without this understanding of the theory about the picture a person viewing the picture would just see a bunch of dots (or tokens, see Figure 1.2 below). This would be like someone not trained in physics trying to make sense out of cloud chamber photographs; or someone not trained in electronics trying to make sense out of an ammeter reading of a plate current; and so on.[2]

Although the pictures that are the subject of this book – spatial maps of parliamentary voting – are *not* art, the concept picture-as-summary is a slippery one that must be used with caution. The boundary line between picture-as-summary and picture-as-art is not as clear as it may appear. For example, consider the most recognized picture in the world – Leonardo Da Vinci's painting, *Mona Lisa*.[3] Why does this painting seem to transcend cultures and national boundaries? Is it the finest portrait ever painted? Is it more important than Picasso's *Les Demoiselles d'Avignon*?[4] The answer of course is obvious -- the *smile*. But why the smile? My opinion is that Leonardo's genius was, figuratively, to flip a coin and have it land on its edge. He managed to paint a facial expression that is exactly on the cusp between a smile and a frown. Consequently, when a person looks at the painting *it does not instantly match* what we recognize as a smile or a frown. So we attend to it longer than we normally would and *we have to think about it*. Hence the fascination.

Figure 1.1
Mona Lisa



Not everyone will agree with my interpretation[5] of *Mona Lisa* but that is not the purpose of my offering it. Clearly, *Mona Lisa* is not a picture-as-summary for most people. Most people see a beautiful painting of a woman with an ambiguous smile. I see a perceptual trick much like the simple figures used by the Gestalt psychologists. It is high art *and* the work of a great scientific mind. This meaning of the picture for me flows from my theory of the picture.

A Theory of Spatial Maps

An Overview

Unlike my *Mona Lisa* theory, the basic space theory of ideology underlying the spatial maps of parliamentary voting analyzed in this book is the end result of the work of a large number of scholars. I am deliberately using the word "theory" broadly and loosely for now. By theory I include: 1) the technical apparatus of the spatial model; 2) a theory of how legislators make decisions; 3) a theory of belief systems (ideology) that is

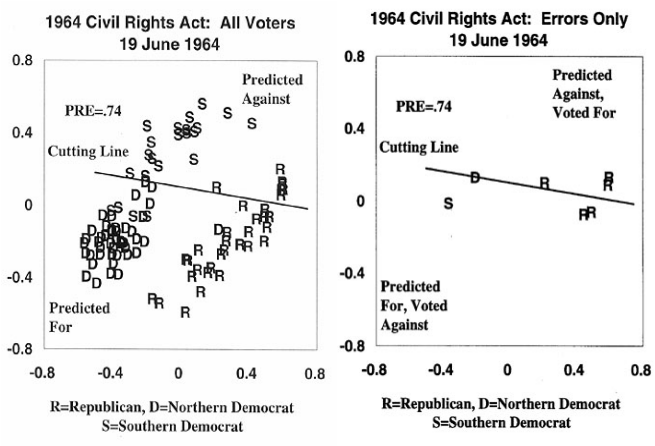
tied to the assumptions of the spatial model and the theory of decision-making; 4) the computer program that embodies (1), (2), and (3) and actually generates the spatial maps; and 5) a substantive understanding of the political system that the parliament or legislature is embedded in. All these are *necessary* for meaning to flow from the spatial map. Simply pushing a matrix of roll call data through a computer program does not itself produce a meaningful picture.

For example, figure 1.2 is a spatial map of the final passage vote of the landmark 1964 Civil Rights Act in the U.S. Senate. The map is from D-NOMINATE (Poole and Rosenthal, 1997). The top panel shows all the Senators and the bottom panel shows just the seven Senators who were errors in the D-NOMINATE analysis. Each Senator's location in the map is a function of all the roll calls the Senator participated in during his/her career. The cutting line is specific to the roll call and divides those Senators who are predicted to vote Yea from those who are predicted to vote Nay. Those Senators who are predicted incorrectly are errors.

Consider just the spatial map of all the Senators. The descriptive labels and the relative positions of the party tokens in the map show that a coalition of Republicans and Northern Democrats voted for the Act and a coalition of Southern[6] Democrats and a few Republicans voted against the Act.[7] The map also shows a clear separation of the Democrats from the Republicans and a sharp division within the Democratic Party. All but 4 of the Southern Democrats are up near the top of the map.

Figure 1.2

Final Passage Vote 1964 Civil Rights Act



My analysis of Figure 1.2 has so far not revealed any information that a sophisticated student of American political history would not already know. For me to go any further requires that I say something about the *structure* of the map. I have to explain what the dimensions represent and explain the relative placement of the D, S, and R tokens on the dimensions. These *explanations* of the structure of the map cannot be based upon the technical apparatus that produced the map. Rather, they must be grounded in the substance of American politics. Furthermore, I have to assume that the reader believes that the technical apparatus that produced the map is reliable and that the reader has a basic understanding of it. Consequently, for me to go further and offer my interpretation of the meaning of the spatial map of the 1964 Civil Rights Act, I need to outline my theory of spatial maps.

Spatial Models of Voting

The spatial maps used in this book rest upon the work of researchers in psychology, economics, and political science. The three fields are equally important to the theory.

In psychology various methods of *multidimensional scaling* (MDS) have been developed during the past 50 years to analyze similarity and preferential choice data. For example, a set of respondents are asked to judge how similar various countries are to each other. MDS methods model these similarities as distances between points representing the countries in a geometric space. These MDS programs are designed to produce a picture-as-summary – to literally summarize a large set of data graphically.[8]

At the same time psychologists were developing MDS, economists and political scientists were developing the spatial theory of voting. In its simplest form the spatial theory of voting can be represented as a map of voters and candidates where the voters vote for the candidate closest to them. In this regard, a spatial map is literally a visual representation of the spatial model of voting. Although Hotelling (1929) and Smithies (1941) are credited with originating the idea, it was the publication of Anthony Downs' *An Economic Theory of Democracy* in 1957 that really established spatial theory as a conceptual tool.

Hotelling studied the logic of the location of a grocery store in a "linear" town; that is, a town strung out along a highway where all the houses are along a single road. It is easy to demonstrate that the optimum location for a grocery store is the *median* of the town (the median minimizes the sum of the walking distances to the store). Hotelling showed that if there are *two* grocery stores, then they will locate adjacent to one another.

Smithies elaborated this model a bit by introducing elastic demand so that people at the edges of town might stop shopping at the store if it moved too far to the center (Downs, 1957, p. 117). Hence, the stores might not converge at the median of the town.

Downs took the Hotelling-Smithies model of spatial competition of stores and applied it to the competition of political parties. Downs assumed that voters were distributed over a dimension – for example, government intervention in the economy – and that political parties played the role of the stores. He derived a large number of classic results from this simple model. For example, if voters vote for the party closest to them on the dimension then the parties will converge to the median voter. Duncan Black (1948, 1958) had earlier derived a similar result for voting in committees.

Although Downs' work is a brilliant *tour de force* it did not present spatial theory in a form that was susceptible to empirical testing. No rigorous mathematical structure was presented from which measuring instruments could be constructed to actually test the theory. The rigorous mathematical structure was provided by the work of Otto Davis, Melvin Hinich, and Peter Ordeshook (Davis and Hinich, 1966; Davis, Hinich, and Ordeshook, 1970). By the early 1970s the mathematical structure of the spatial theory was largely completed. The dimensions of the space represented issues/policies. Each voter had a position on each issue/policy and this vector of positions was the voter's *ideal point* in the space. Each voter also had a utility function centered on her ideal point that assigned a utility to each point in the space. The further a point was from the voter's ideal point the lower the utility. Each candidate also had a position on each position and therefore was represented as a point in the space. Each voter then voted for the candidate for whom she had the highest utility. In the context of parliamentary voting the model is

exactly the same only policy outcomes are now the choices rather than candidates for public office.

This early version of the spatial theory of voting did not allow for "error" on the part of voters. That is, voting was deterministic. Voters had ideal points and voted for the candidate closest to them in the policy space. Later more realistic assumptions about voters' decision rules allowed for probabilistic voting.[9] Nevertheless, this version of spatial theory could at least be investigated empirically.

Psychometrics and Tests of Spatial Theory

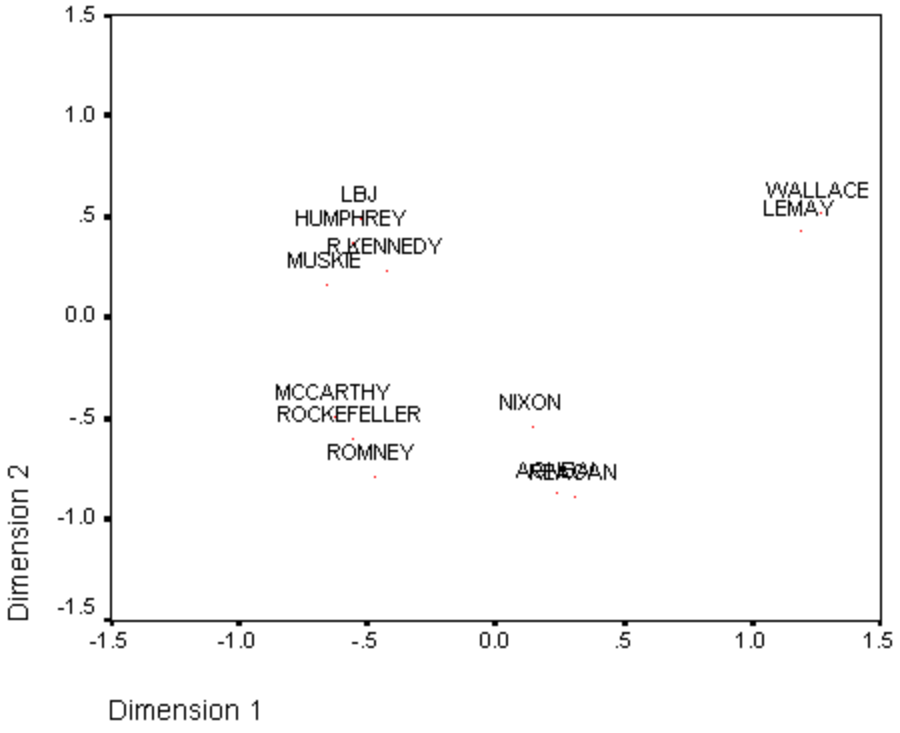
In order to test the spatial theory of voting you need data from voters about how "far" they are from candidates. The first comprehensive test of spatial theory was by Cahoon (1975) and Cahoon et al. (1978) using candidate feeling thermometers. Beginning in 1968 feeling thermometers were included in the NES surveys. A feeling thermometer measures how warm or cold a person feels towards the stimulus and the measure ranges from 0 – very cold and unfavorable opinion – to 100 – very warm and favorable opinion with 50 being a neutral point. In 1968 respondents were asked to give feeling thermometer ratings to the presidential candidates George Wallace, Hubert Humphrey, and Richard Nixon, along with their vice presidential running mates and six other political figures.[10] These thermometer scores can be interpreted as distances between the respondent's ideal point and the spatial position of the candidates. For example, if a respondent gave Humphrey a 100, Nixon a 40, and Wallace a 0, then she was likely to be very close to Humphrey, very far from Wallace but closer to Nixon than to Wallace.

Cahoon constructed a statistical model of the thermometer scores based on the spatial theory of voting and found that a simple two-dimensional spatial map largely accounted for the observed thermometer scores. He was able to place both the candidates and the respondents in the map such that his predictions of how the respondents would vote closely matched their actual voting choices.

Figure 1.3 shows the configuration of presidential candidates similar to that estimated by Cahoon. The horizontal dimension is a liberal (on the left) to conservative (on the right) dimension and the second dimension is a Democrat (towards the top) to Republican (towards the bottom).

Figure 1.3

1968 Presidential Candidate Configuration



Cahoon was the first to test the spatial theory of voting using thermometer scores but he was not the first to construct spatial maps from thermometer scores. Herbert Weisberg and Jerrold Rusk (1970) used the MDS procedure developed by Kruskal (1964a,b) to recover a candidate configuration from the candidate-by-candidate correlation matrix computed across the respondents. They did not estimate the respondent locations.[11]

Although the spatial maps produced by Cahoon and Weisberg and Rusk are essentially the same and are both pictures-as-summary, they have very different theoretical foundations. The MDS procedures developed by psychologists were intended to help answer questions of importance to psychologists. Namely, given a set of judged similarities between objects (nations, colors, types of crime, emotional states, etc.), MDS procedures were used to uncover underlying psychological dimensions or as a tool to formulate a convincing description of the data. For example, two dimensions – communist/noncommunist and developed/underdeveloped – were found to underlie similarity judgments of nations (Kruskal and Wish, 1979). In contrast, the spatial theory of voting is a *theory of behavior* that states that *if* a set of assumptions holds, *then* voters should behave in a certain way *and* we should observe certain types of outcomes. It is a theory that makes predictions that can be *tested*.

Although the theoretical foundations are different, as a practical matter the MDS procedures developed by the psychologists are very similar in form to procedures developed to test spatial theory. A full blown test of spatial theory like that performed by Cahoon estimates ideal points for the voters and points for the candidates in a spatial map such that the distances between the voters and candidates in the map are as close as

possible to the original data. For example, the thermometers range from 0 to 100. Cahoon transformed these into distances by subtracting the thermometer value from 100. Finding points for the voters and points for the candidates that reproduce these distances is known as an *unfolding analysis* in psychology (Coombs, 1964). Techniques to perform unfolding analyses were developed by psychologists in the 1950s and 1960s (Chang and Carroll, 1969; Kruskal, Young, and Seery, 1973) and Cahoon's method is also an unfolding methodology. Later Rabinowitz (1976), Poole and Rosenthal (1984a), and Brady (1990) developed unfolding procedures that they applied to thermometer scores.[12]

Analyses of roll call data during this period did not attempt to unfold the roll call data. Rather, factor analysis and cluster analysis was used to analyze correlation/covariance matrices computed between members across the roll calls (MacRae, 1958, 1970).[13] Similarly, Hoadley (1980) analyzed the emergence of political parties in the early Congresses by computing agreement scores between members computed across the roll calls and used Kruskal's MDS procedure to recover configurations of members of Congress. Weisberg (1968) in his analysis of roll call voting showed that one dimensional spatial voting would look much like the responses to a classic Guttman scale (Weisberg, 1968). However, he did not attempt the joint estimation of a spatial map of both the roll calls and legislators.

Why So Few Dimensions? Psychometrics and Multidimensional Scaling

The early tests of spatial theory produced two important puzzles. First, the number of dimensions of the spatial maps appeared to be three or less and in most

applications two dimensions seemed to adequately account for the data. And second, contrary to the predictions of spatial theory, the candidates did not appear to have converged near the median voter or anywhere near the center of the space. This latter problem was later remedied by advances in the theory by a number of scholars.[14] For example, if candidates are motivated by their policy preferences then they may not converge to the center of the space.

With the benefit of hindsight, the finding of low dimensionality should have been no surprise. Low dimensional maps were a common result of MDS applications in psychology by the early 1970s. For example, the experimental data on the perception of color, the perception of sound, the similarity of Morse code signals, similarity of nations, relatedness among societal problems (war, poverty, crime, etc.), perceived association of psychological traits (honest, helpful, sincere, tolerant, etc.), and similarity of diseases, all fit simple two-dimensional maps.[15] In addition, psychologists had found that people are limited in their ability to perceive distinct objects (Miller, 1956).

Within political science, survey researchers had shown by the late 1960s that most voters did not have highly structured attitudes about politics (Campbell, Converse, Miller, and Stokes, 1963; Nie, Verba, and Petrocik, 1978). Only a small slice of Americans could be said to have *coherent* sets of issue positions. In 1964 Philip Converse published his seminal essay "The Nature of Belief Systems in Mass Publics" that drew heavily upon his experience with these surveys. Converse defined a belief system "…as a configuration of ideas and attitudes in which the elements are bound together by some form of constraint or functional interdependence" (p. 207). Constraint means that issues are interrelated or bundled and that ideology is fundamentally *the knowledge of what-*

goes-with-what.[16] From an observer's point of view, the knowledge of one or two issue positions makes the remaining positions very predictable. The simplest of all is the *liberal-moderate-conservative* continuum so familiar to journalists and the political cognoscenti. "The efficiency of such a yardstick in the evaluation of events is quite obvious. Under certain appropriate circumstances, the single word 'conservative' used to describe a piece of proposed legislation can convey a tremendous amount of more specific information about the bill – who probably proposed it and toward what ends, who is likely to resist it … its long-term social consequences…and … how the actor himself should expect to evaluate it…" (p. 214). In contemporary American politics the knowledge that a politician opposes raising the minimum wage makes it virtually certain that the politician opposes universal health care, opposes affirmative action, and so on. In short, a *conservative* and almost certainly a Republican.

Although forty years of empirical work by political scientists has shown that the American public is not *strictly* ideological in that most people do not have *highly structured* belief systems, they also are not "ideologically innocent" either (Feldman and Zaller, 1992). Ideological consistency, that is, the degree to which issue positions are coherently generated by a few underlying basic issues such as liberal-conservative, clearly varies. It is almost certainly a top-down phenomenon. Political elites are more ideologically consistent than the mass public and it is quite likely that this has an impact on how issues are "packaged" (Hinich and Munger, 1997, ch. 9). Hence, members of national legislatures such as the U.S. Congress should exhibit highly structured belief systems.

The Breakthrough: The Two-Space Theory

If the voters have highly structured belief systems, then within the spatial theory of voting this means that their issue positions lie on a low-dimensional plane through the issue space because attitudes across the issues are *constrained*. The presence of constraint means that a voter's positions on a variety of specific issues can be captured by her position on one or two fundamental dimensions such as liberalism/conservatism. This implies *two* spaces – one with a few fundamental dimensions and a second high-dimensional space representing all the distinct issues. For example, suppose there are s fundamental dimensions, p voters, and q issues where s < q. Let **X** be the p by s matrix of ideal points of the p voters on the s dimensions and let **Y** be the p by n matrix of voters' ideal points on the n issues. The presence of constraint means that the voters' positions on the fundamental dimensions **X** generate all the issue positions **Y**; that is, **XW = Y** where the s by q matrix **W** maps the fundamental dimensions onto the issue dimensions.[17] Under this interpretation, the low dimensional maps produced by the various scaling procedures discussed above show the low-dimensional space underlying individuals' evaluations -- the "**X** space" -- not the multidimensional issue space -- **Y**.

This two-space theory was stated as a conjecture by Cahoon, Hinich, and Ordeshook (1976) who dubbed the low dimensional space a *basic space* and the high dimensional space an *action space* containing all "…contemporary political issues [and] government policies…" (Ordeshook, 1976, p. 308)[18]. Hinich and his colleagues then developed the theory in detail including how one space mapped into the other (Hinich and Pollard, 1981; Enelow and Hinich, 1984). They labeled the dimensions of the low dimensional space the *predictive dimensions*. More generally, these are latent or

evaluative dimensions and in political science work are commonly referred to as *ideological dimensions* (Hinich and Munger, 1994; 1997). I will refer to these as *basic dimensions* in this book.[19]

The 1964 Civil Rights Act

The basic space theory of ideology outlined above permits a parsimonious interpretation of the spatial map of the final passage vote of the 1964 Civil Rights Act shown in Figure 1.2. There are two basic dimensions. The primary dimension is indeed the liberal-moderate-conservative dimension epitomized by voting on the fundamental issue of the role of government in the economy. The second dimension captures the conflict over race and civil rights and it emerged during the latter part of the New Deal when, in the wake of the 1936 elections, northern Democrats heavily outnumbered southern Democrats in Congress. Many of the programs initiated during the subsequent Second New Deal were not to the liking of the south. Voting on minimum wages in 1937 and 1938 followed by voting during World War II on the poll tax and voting rights in the armed forces helped to split the Democratic party into two distinct regional wings (Poole and Rosenthal, 1997; McCarty, Poole, and Rosenthal, 1997). Voting in Congress became two dimensional in order to differentiate northerners from southerners on civil rights and related votes. With the passage of the 1964 Civil Rights Act, the 1965 Voting Rights Act, and the 1967 Open Housing Act, this second dimension slowly declined in importance and is now almost totally absent. Race related issues – affirmative action, welfare, Medicaid, subsidized housing, etc. – are now largely questions of *redistribution*. Voting on these issues along with more symbolic issues like hate-crimes legislation, now

take place along the liberal-conservative dimension and the old split in the Democratic Party between North and South has largely disappeared.[20] Voting in Congress is almost purely one-dimensional – a single dimension accounts for almost 90 percent of roll call voting choices in the 104th, 105th, and 106th Congresses – and the two parties are increasingly polarized.[21]

Note that I used *both* the basic dimensions and specific issue dimensions to interpret the 1964 Civil Rights Act vote. The basic dimensions cannot be understood unless *all* the roll calls are classified by their substantive content. The overall fit of a set of roll calls with same issue content can be used to determine the extent to which a specific issue dimension is captured by the basic space.

The Rest of This Book

In the next two chapters I develop the geometry of roll call voting. Legislators are represented as points and the roll calls are represented as lines/planes that pass through the legislators dividing them into those that vote Yea and those that vote Nay. The configuration of legislator points determines the locations of the roll call lines/planes and the configuration of roll call lines/planes determines the locations of the legislator points. Chapter 2 deals with *perfect voting*, that is, voting with no errors by the legislators. This allows me to show the inherent limits that the geometry places upon any scaling procedure. For example, in one dimension with perfect voting, the legislators are identified only up to a rank ordering. In more than one dimension legislators are only identified up to polytopes (regions bounded by cutting lines/planes).

In Chapter 3 I introduce error via the *random utility model* and show how the geometry interacts with standard probabilistic models of voting. The geometry of roll call voting shown in Chapter 2 powerfully restricts the role of error in the model. Because legislators are identified only up to polytopes and the distances between these polytopes are not *necessarily* linear in the number of cutting lines/planes between them, error, *which is manifest only in the changes of Yeas to Nays and vice versa in a legislator's voting pattern*, cannot be guaranteed to be symmetric with direction in the space.

Chapter 4 shows how the various scaling procedures for analyzing roll call data developed over the past 50 years are grounded in the geometry discussed in chapters 2 and 3. This discussion enables me to address practical problems of estimation. How many legislators and roll calls do you need to estimate a spatial map that makes sense? How many dimensions should you estimate? Once you have the dimensions, how do you figure out what they mean? I will also show how things can go wrong with the various scaling procedures and the warning signs to look for in the spatial maps. I will also apply various scaling methods to the same legislature to illustrate their uses and misuses and their advantages and disadvantages.

Chapter 5 discusses more advanced topics. How roll call data matrices can be split into various pieces to estimate more elaborate spatial models (for example, treating members who exit the legislature differently from those who continue to the next session). How to combine multiple legislatures so that dynamic spatial models can be estimated. Finally, how to construct animated graphical displays to study the dynamics of legislative voting. I will also outline some basic programming "tricks" that I have

learned over the past 30 years that may be useful to those who wish to “roll their own” scaling program.

I conclude in chapter 6 with some thoughts about the scientific role and meaning of spatial maps of voting in general.

Chapter 1 Notes

[1] I borrowed this analogy from Jordan Ellenberg who used it in an article about my political polarization research with Howard Rosenthal (Ellenberg, 2001).

[2] Philosophers of science have explored this topic in great depth. Kuhn (1962/1996, 187-191) has a nice discussion of the training of physicists that illustrates the shift in meaning between amateur and specialist.

[3] Leonardo began painting *Mona Lisa* about 1503 and worked on it for many years. Francis I of France bought the portrait but let Leonardo keep it until his death in 1519.

[4] *Les Demoiselles d'Avignon* was painted in 1907 and is considered a landmark in modern art. It marked the beginning of Picasso's long Cubist period.

[5] For a more traditional interpretation of *Mona Lisa* see Gombrich (1978, pp. 227-229). He also emphasizes the role of Leonardo the scientist in the construction of *Mona Lisa*.

[6] Throughout this book the South is defined as the 11 states of the Confederacy plus Kentucky and Oklahoma.

[7] The vote was 73 Yea and 27 Nay. There were 67 Democrats voting and they split 46 Yea and 21 Nay (Northern Democrats split 43 Yea and 1 Nay; Southern Democrats split 3 Yea and 20 Nay). The 33 Republicans split 27 Yea and 6 Nay.

[8] Two eminent psychometricians Ingwer Borg and Patrick Groenen state this very clearly: "The main reason for doing [MDS] is that one wants a graphical display of the structure of the data, one that is much easier to understand than an array of numbers…" (Borg and Groenen, 1997, p.vii).

[9] For example, the standard formulation is to make the probability that a voter chooses the closest candidate a function of both the closeness of the candidate to the voter and a random draw from an error distribution. See McFadden (1976) for a survey of random utility models.

[10] In 1968 respondents were asked to give feeling thermometer ratings to 12 political figures: George Wallace, Hubert Humphrey, Richard Nixon, Eugene McCarthy, Ronald Reagan, Nelson Rockefeller, Lyndon Johnson, George Romney, Robert Kennedy, Edmund Muskie, Spiro Agnew, and Curtis LeMay. The NES survey was conducted after Robert Kennedy's assassination in June, 1968. This obviously affected the ratings Kennedy received.

[11] The 1968 set of thermometer scores have been analyzed by a variety of scaling techniques. See Wang et al. (1975); Rabinowitz (1976); Poole and Rosenthal (1984a); for examples.

[12] Technically, Brady estimates the *distribution of the voters* rather than the ideal points of the voters.

[13] MacRae proposed the model of roll call voting that Howard Rosenthal and I implemented as NOMINATE -- namely, ideal points for legislators and two policy outcomes per roll call, one for Yea and one for Nay. However MacRae did not attempt to estimate this unfolding model.

[14] For example, Wittman (1977, 1983), Palfrey (1984), Bernhardt and Ingberman (1985), Calvert (1985), Austen-Smith (1987), Morton (1987), Aldrich and McGinnis (1989), Harrington (1992), and Spector (2000).

[15] See Ekman (1954) and Borg and Groenen (1997) for the color circle example; Shepard (1963) for sound and morse code examples; Wish (1971) for nations; Wish and Carroll (1974) for societal problems;

Rosenberg, Nelson and Vivekananthan (1968) for traits; and D'Andrade, Quinn, Nerlove, and Romney (1972) for disease.

[16] As Hinich and Pollard (1981) point out, it is not necessary that the person influenced by the ideology know *why* what-goes-with-what. Rather, words like "liberal" and "conservative" are best understood as labels that have become attached to certain consistent patterns of political behavior. (See Poole, 1988, for an elaboration of this point).

[17] Alternatively, the issue positions can be viewed as being mapped onto the fundamental dimensions; namely: $\mathbf{X} = \mathbf{YW}$ where the q by s matrix $\mathbf{W}$ maps $\mathbf{Y}$ into $\mathbf{X}$. This is almost certainly true of *new* issues (Poole and Rosenthal, 1997, ch. 5).

[18] Ordeshook (1976) proposed a model with *three* spaces – a common underlying preference space, an action space, and a third space identical to the action space only it records the positions of the candidates. It is not clear that Ordeshook's "common underlying preference space" is of low dimensionality but in a conversation with me in November 1976, he explicitly outlined in detail the basic space/action space model used in this book.

[19] My use of the label "basic dimensions" is also motivated by the fact that Horst (1963) refers to the singular value decomposition of a rectangular matrix as the *basic structure* of a matrix. Roll call matrices are inherently rectangular and any method that estimates parameters for legislators and parameters for roll calls is performing a form of matrix decomposition. Hence the connection.

[20] See Carmines and Stimson (1989) for a brilliant discussion of how race has transformed American party politics since the end of World War II.

[21] Polarization is analyzed by Poole and Rosenthal (1984b; 1997), King (1998), and McCarty, Poole, and Rosenthal (2001).